

CHAPTER 2

A CUBIK ORIENTATION

Before developing a strategy for restoring the cube, it helps to study the cube a little while. What can we observe about the cube that may help us with the solution? What simple terminology and notation will describe the pieces and movements of the cube? Figure 2-1 gives a summary of the terminology and notation to be developed in this chapter.

1. CUBIES AND CUBICLES

Looking at the cube as a whole, at first glance, it appears to be made up of $3 \times 3 \times 3 = 27$ cubies in three layers, each layer being a three-by-three square of small cubies. However, it is only possible to see the outside of the cube, so that only 26 cubies can be seen. The one in the center is only imaginary. Also, all that we can see of each of the 26 visible cubies are the colored facelets which combine to form the six faces of the entire cube. Each face of the cube is made up of nine such facelets. Thus there are $6 \times 9 = 54$ facelets on the cube.

Look now at the cubies which make up the cube. Notice that the cube has three types of cubies. Some cubies have three visible

SUMMARY OF TERMINOLOGY AND NOTATION

Terminology	Definition or Abbreviation						
Cubies	The small cube pieces which make up the whole cube.						
Cubicles	The spaces occupied by cubies.						
Facelets	The faces of a cubie.						
Types of Cubies: Corner, Edge, and Center	A corner cubie has three facelets. An edge cubie has two facelets. A center cubie has one facelet.						
Home Location - of a cubie	The cubicle to which a cubie should be restored.						
Home Position - of a cubie	The orientation in the home location to which a cubie should be restored.						
Positional Names for Cube Faces	<table style="margin-left: auto; margin-right: auto;"> <tr> <td>Up</td> <td>Down</td> </tr> <tr> <td>Right</td> <td>Left</td> </tr> <tr> <td>Front</td> <td>Back</td> </tr> </table>	Up	Down	Right	Left	Front	Back
Up	Down						
Right	Left						
Front	Back						
Notation for Cubicles - shown in <i>italics</i>	Lower case initials. For example, <i>uf</i> denotes the Up-Front edge cubicle.						
Notation for Cubies - shown in <i>italics</i>	Upper case initials. For example, <i>URF</i> denotes the cubie whose home position is in the Up-Right-Front corner						
Notation for Face Turns - shown in BLOCK CAPITAL LETTERS	The initials, U, F, R, D, B, and L denote clockwise quarter turns. U^{-1} , F^{-1} , R^{-1} , D^{-1} , B^{-1} , and L^{-1} denote counter-clockwise quarter turns. U^2 , F^2 , R^2 , D^2 , B^2 , and L^2 denote half turns.						
Moving the Whole Cube	\mathcal{U} , \mathcal{F} , \mathcal{R} , \mathcal{D} , \mathcal{B} , and \mathcal{L} denote clockwise turns of the whole cube behind the indicated face.						

Figure 2-1

facelets as indicated in Figure 2-2. These are called *corner pieces*. There are eight corner pieces corresponding to the eight corners of the cube. Other cubies have only two visible facelets as indicated in Figure 2-3. These cubies fill in the space along an edge between two corner pieces. Therefore, they are called *edge pieces*. There are twelve edge pieces, one on each of the twelve edges of the cube. The third type of cubie has only one visible facelet. This facelet, as shown in Figure 2-4 is in the middle of a face. Thus these cubies are called *center pieces*. There are six center pieces corresponding to the six faces of the cube.

THE CORNER CUBIES, SHADED, HAVE THREE FACELETS

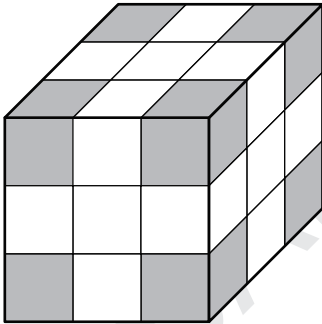


Figure 2-2

THE EDGE CUBIES, SHADED, HAVE TWO FACELETS

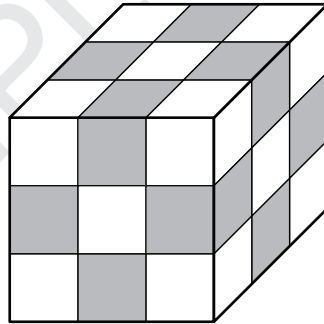


Figure 2-3

THE CENTER CUBIES, SHADED, HAVE ONE FACELET

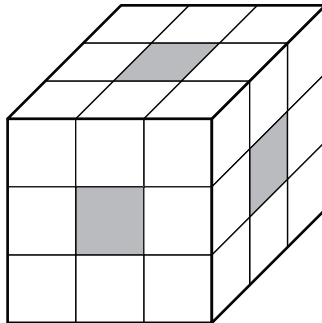


Figure 2-4

By rotating different faces of the cube, the cubies can be moved about. Each cubie moves to the location vacated by another cubie. These locations are called *cubicles*. The locations occupied by corner cubies are corner cubicles and the locations occupied by edge cubies are edge cubicles. Observe that no matter how faces are rotated, the corner pieces always move from one corner cubicle to another corner cubicle and the edge pieces always move from one edge cubicle to another edge cubicle. Rotating a face never moves a center cubie from one face to another. The center pieces have a fixed location relative to the other center pieces. They can only be spun in place. This is a particularly important observation, because it shows the following:

The color of the center piece of any face defines the only color to which that face of the cube can be restored.

For each center piece the color of the opposite center piece never changes. Furthermore, if two opposite center pieces are placed in the positions of north and south poles respectively, then the sequential order of the other four center pieces around the equator is always the same.

Since the center cubie of each face determines the only color to which that face can be restored, we can also define the one and only cubicle in which each cubie can be placed to restore the cube. For example, if the two facelets of an edge cubie are orange and green, then that piece must be placed in the unique edge cubicle between the orange center piece and the green center piece as shown shaded in Figure 2-5. Furthermore, the cubie must be placed in that cubicle so that its orange facelet is next to the orange center piece and the green facelet is next to the green center piece.

Similarly, if the three facelets of a corner cubie are orange, green, and white then, to restore that cubie, it must be placed in the corner cubicle where the orange face, the green face, and the white face meet – shaded in Figure 2-6. Furthermore, its orange, green, and white facelets must be on the orange, green, and white faces respectively.

For each edge and corner cubie in the cube, the unique cubicle to which it must be restored is called the *home location* for that

AN EDGE HOME POSITION

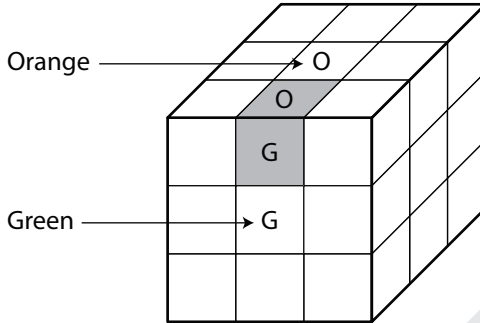


Figure 2-5

cubie. When a cubie is in its home location and its facet colors match the colors of the center pieces on each face, then the cubie is said to be in its *home position*.

It is possible for a cubie to be in the cubicle of its home location without being in its home position. A corner piece in this condition is said to be *twisted* in its home location. An edge piece in this condition is said to be *flipped* in its home location. Figure 2-7 shows a twisted corner cubie and a flipped edge cubie. Thus, each corner and edge cubie has a unique home location and in that cubicle it has a unique placement which puts it in its home position.

A CORNER HOME POSITION

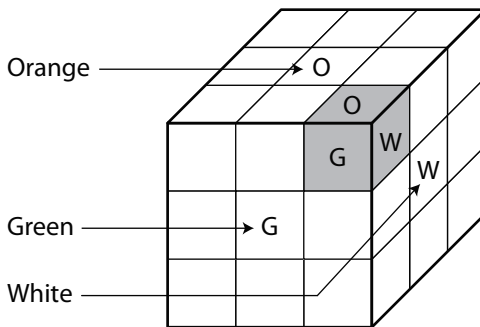


Figure 2-6

TWISTED AND FLIPPED CUBIES

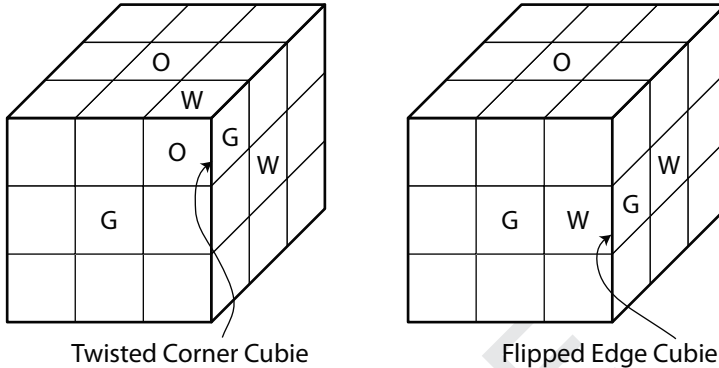


Figure 2-7

EXERCISES:

2.1-1 How many of the 54 facelets of the cube are

- a) facelets of corner cubies?
- b) facelets of edge cubies?
- c) facelets of center cubies?

2.1-2 At how many locations can an edge cubie be placed so that the colors of both of the two adjacent center cubies are different from both colors on the facelets of that edge cubie?

2.1-3 In what cubicle can a corner cubie be placed so that none of the center cubies adjacent to that cubicle has the color of any of the three facelets of that corner cubie? Describe the cubicle location relative to the home position of the cubie.

2. ORIENTATION BASED ON THE CENTER FACELETS

To discuss the movements of the cube and its cubies, we need to establish a terminology and notation. The most important quality of any terminology and notation is that it be accepted and used in the same way by all the people who need to communicate about the subject. There is one terminology and notation that has been accepted internationally by most students of the cube. It was devised by David Singmaster of Polytechnic of the South Bank, London, England. That terminology is the one we use in this book.

Many people who work with the cube have developed their own terminology and notation for cube movements. Perhaps you already have a notation of your own. Different people's notations reflect their diverse ways of approaching the cube problems. Even if one of these were better than the one we use in this book, we would not want to change because the one presented in this book is already the most widely used.

We want to be able to discuss how cubies move from cubicle to cubicle in the cube. To do this, we need to describe the location of each cubicle. Since rotating a face never changes the location of the center cubies, it is natural to use these center pieces as reference points to describe the locations of the cubicles.

It is tempting to name each face by the color of the center facelet. However, different manufacturers use different colors and even the same manufacturer does not always keep the same relative placement for coloring the faces of different cubes. In discussions among people with such different cubes, using colors to identify cubies, cubicles, and the movements of the cube may be more confusing than helpful.

To avoid the confusions caused by different cube colorings, each of the faces is named based on its position relative to the person holding the cube. Singmaster has chosen the following six names for the six faces: Front, Back, Right, Left, Up, and Down. These faces are designated by their initials as follows:

Front = F
Back = B
Right = R
Left = L
Up = U
Down = D

It is very convenient to be able to abbreviate the names of the faces by their initials. Singmaster chose these names to avoid the ambiguities presented by the initials of some of the logical English words for the faces; Right/Rear and Back/Bottom.

To designate a face on your own cube as the Up face, choose any center cubie which you like to be the Up-face center piece. From

then on the pattern of all facelets which appear at any moment on the same face as that chosen center-piece facelet will constitute the Up face. The Up layer is the set of all cubies which have a facelet on the Up face. The color of the Up-face center facelet is the color to which the Up face must be restored.

After choosing a center cubie for the Up face, you can choose any of four other center cubies for the Front-face center piece. After you have chosen colors for the center facelets of the Up face and the Front face, then all the other center facelet colors are fixed by the way the manufacturer put the colors on the cube. Thus we now have matched each of the six faces of the cube with a different one of the six positional names Up, Front, Right, Down, Back, and Left.

This correspondence between the cube faces and positions in space defines an orientation of the cube. The *orientation* of an object is its position relative to an agreed-upon point of reference. In many cases the point of reference may be moveable. For example, we can use North or the "front" of a car or even the direction in which you are looking as points of reference and describe the orientation of other objects accordingly. Thus if a car has turned over in an accident we say that the part of the car which is on the bottom is the "top" of the car. The same is true after we have defined an orientation for the cube. We may turn the cube over to look at the Down face. This may temporarily put the Up face on the bottom and the Front face in the back. But this does not change the orientation of the cube. The color which we assigned to the Up face remains the Up-face color and the color which we assigned to the Front is still the Front-face color. However, we can reorient the cube. We say that we have reoriented the cube whenever we assign a different color to one of the six positional names. Thus if you turn the cube around just to look at the back then return it, that does not reorient the cube. But, we will find times when it is useful to turn it around and keep the color that was the Backface color in front and call it the Front-face color. That renaming process is called *reorienting* the cube.

We also use the positional names of the faces to identify the cubicles, cubies, and facelets. For example, the edge cubicle between the Up-face center piece and the Frontface center piece

can be called either the Up-Front cubicle or the Front-Up cubicle. The edge cubie which has one Down-colored facelet and one Right-colored facelet is the Down-Right cubie – or also the Right-Down cubie. The home location of the Left-Back-Up corner cubie is the corner cubie where the Up face, the Left face and the Back face meet – the Up-Left-Back cubie. This cubie is in its home position when it is in its home location with the Up-colored facelet on the Up face.

EXERCISES: (* indicates harder.)

2.2-1 After choosing an Up-face color, how many different choices are possible for the Front-face color?

2.2-2 Which of the following statements are true and which are false?

- a) When the cube is restored so that each face is a solid color, the colors on opposite faces will always be the same.
- b) When the positions of the center pieces of two adjacent faces are named then the positional names of all the center pieces are fixed.
- c) When the positions of any two center pieces are named then the positional names of all the center pieces are fixed.

2.2-3 * How many different orientations of the cube are possible?

3. NOTATION FOR ABBREVIATIONS

We will abbreviate the six positional names by their initials. To distinguish between the symbols for cubicles and the symbols for cubies, we will use lower case italics for the cubicles and upper case italics for the cubies. Thus the four edge cubicles in the Up layer – shaded in Figure 2-8 – are either denoted *uf*, *ul*, *ub*, and *ur*, or denoted *fu*, *lu*, *bu*, and *ru*. The four cubies whose home locations are the cubicles *uf*, *ul*, *ub*, and *ur* are denoted either by *UF*, *UL*, *UB*, and *UR* or by *FU*, *LU*, *BU*, and *RU*. The order of the initials of the cubie in the cubicle is used to indicate which facelet is on which face. For example, saying that the *BU* cubie is in the *ur* cubicle means that the Back-colored facelet is on the Up face and the Up-

EDGE CUBICLES IN THE UP LAYER

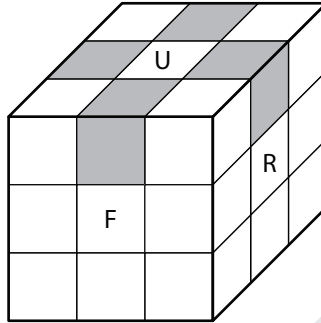


Figure 2-8

colored facelet is on the Right face. There are twelve edge cubicles and twelve edge cubies denoted as follows:

Cubicles	Cubies
<i>uf</i> or <i>fu</i>	<i>UF</i> or <i>FU</i>
<i>ul</i> or <i>lu</i>	<i>UL</i> or <i>LU</i>
<i>ub</i> or <i>bu</i>	<i>UB</i> or <i>BU</i>
<i>ur</i> or <i>ru</i>	<i>UR</i> or <i>RU</i>
<i>rf</i> or <i>fr</i>	<i>RF</i> or <i>FR</i>
<i>fl</i> or <i>lf</i>	<i>FL</i> or <i>LF</i>
<i>lb</i> or <i>bl</i>	<i>LB</i> or <i>BL</i>
<i>br</i> or <i>rb</i>	<i>BR</i> or <i>RB</i>
<i>df</i> or <i>fd</i>	<i>DF</i> or <i>FD</i>
<i>dl</i> or <i>ld</i>	<i>DL</i> or <i>LD</i>
<i>db</i> or <i>bd</i>	<i>DB</i> or <i>BD</i>
<i>dr</i> or <i>rd</i>	<i>DR</i> or <i>RD</i>

The corner cubicles and cubies are similarly denoted by their facelets. The corner cubicle shaded in Figure 2-9 is the *urf* cubicle. The *urf* cubicle is the home location of the *URF* cubie. The facelets of a corner cubicle or of a corner cubie are always written in a clockwise order as shown in Figure 2-10. The clockwise order in this case is determined by looking along a diagonal line from the outside corner of the cube to the center of the cube. Thus, the three clockwise designations *URF*, *RFU*, and *FUR* all refer to the same piece whose

home location is the *urf* cubicle. That same cubicle is also denoted by *rfu* or *fur*. The counter-clockwise designations, *UFR*, *FRU*, and *RUF* and *ufr*, *fru*, and *ruf* are never used. Again the order of the initials is used to show the orientation of a cubie in a cubicle. For example, saying the *BLD* cubie is in the *urf* cubicle means that its Back-colored facelet is on the Up face, its Left-colored facelet is on the Right face, and its Down-colored facelet is on the Front face. There are eight corner cubicles and eight corner cubies denoted as follows:

Cubicles	Cubies
<i>urf, rfu, or fur</i>	<i>URF, RFU, or FUR</i>
<i>ufl, flu, or luf</i>	<i>UFL, FLU, or LUF</i>
<i>ulb, lbu, or bul</i>	<i>ULB, LBU, or BUL</i>
<i>ubr, bru, or rub</i>	<i>UBR, BRU, or RUB</i>
<i>dfr, frd, or rdf</i>	<i>DFR, FRD, or RDF</i>
<i>dlf, lfd, or fdl</i>	<i>DLF, LFD, or FDL</i>
<i>dbl, bld, or ldb</i>	<i>DLB, BLD, or LDB</i>
<i>drb, rbd, or bdr</i>	<i>DRB, RBD, or BDR</i>

We use the order of the facelets when describing the movement of pieces resulting from a sequence of face turns. We may say that the piece in the *ufl* cubicle moves to the *drb* cubicle, and write

$ufl \rightarrow drb$.

THE *urf* CUBICLE

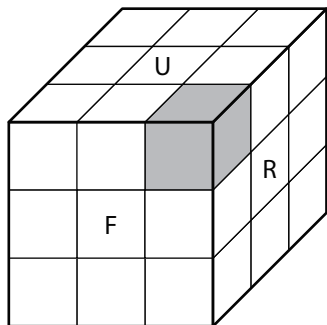
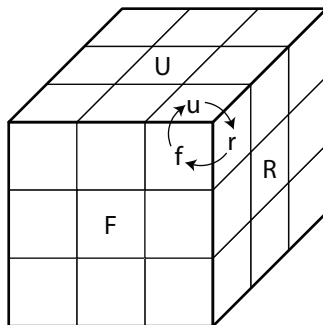


Figure 2-9

DESIGNATING CORNER FACELETS



Facelets are designated in clockwise order.

Figure 2-10

ALL CUBIES IN THEIR HOME POSITIONS

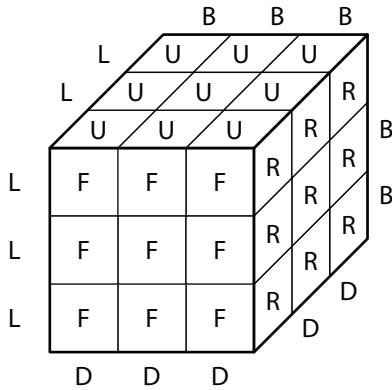


Figure 2-11

A SCRAMBLED CUBE

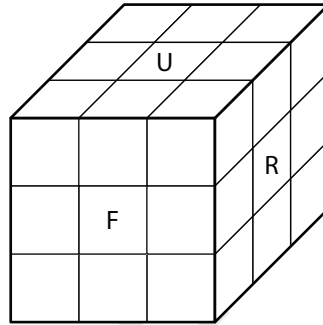


Figure 2-12

By this we mean that the facelets of the corner cubie in the *ufl* cubicle move so that the facelet from the Up face moves to the Down face, and the other facelets move with it so that

$$\begin{aligned} u &\rightarrow d \\ f &\rightarrow r \\ l &\rightarrow b. \end{aligned}$$

When we write

$$rb \rightarrow bu$$

we mean that the edge cubie in the *rb* cubicle moves so that the facelet starting on the Right face moves to the Back face in the *bu* cubicle.

Similarly, we show a clockwise twist of a corner cubie in the *urf* cubicle by writing

$$urf \rightarrow rfu,$$

or the flip of an edge cubie in the *fl* cubicle by writing

$$fl \rightarrow lf.$$

APPLYING THE MOVE F TO A RESTORED CUBE

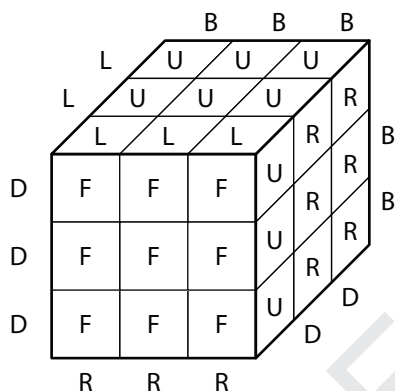


Figure 2-13

We illustrate the positions of cubies in a cube with a diagram of the type shown in Figure 2-11 which shows all cubies in their home positions. Each initial indicates that the color of the facelet in which it appears is the color of the center cubie with that initial. The initials around the outside of the cube indicate the color of the unseen facelet along the edge adjacent to the initial. No initial is placed in a facelet when its color is unknown. Thus Figure 2-12 illustrates a scrambled cube. As the cube is gradually restored, more and more initials appear.

Once you have established an orientation for your cube, you can move cubies about from cubicle to cubicle by rotating any of the six faces of the cube. These six face rotations are denoted by the initials in block capital letters

U, D, F, B, R, and L.

A single initial indicates a clockwise quarter turn of the corresponding face. Thus Figure 2-13 shows the cube after the move F has been applied to the cube in its starting state shown in Figure 2-11. The direction of a clockwise quarter turn for any face rotation is defined as shown in Figure 2-14 by viewing that face from that side of the cube. A half turn of any face is two quarter turns of that face. So we use the notation

U^2 , D^2 , F^2 , B^2 , R^2 , and L^2

to denote half turns of the six faces. The symbol U^2 is pronounced “U squared.” Counter-clockwise quarter turns are denoted by

$$U^{-1}, D^{-1}, F^{-1}, B^{-1}, R^{-1}, \text{ and } L^{-1}.$$

The symbol U^{-1} is pronounced “U inverse.”

In recording a sequence of moves, we list the moves from left to right. Thus FR means apply F first and then apply R. Figure 2-15 shows the result of applying the sequence FR to a cube in its starting state as shown in Figure 2-11. Any sequence of moves is called a *process*. Sometimes parentheses are placed around a sequence of moves within a process only to emphasize that those moves are related. Such parentheses can always be ignored without changing the overall process.

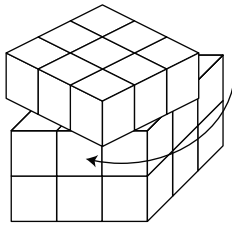
To indicate that a particular process moves cubies from one cubicle to another cubicle, we list the cubicle movement followed by the process. For example, we write

$$uf \rightarrow ru: FR$$

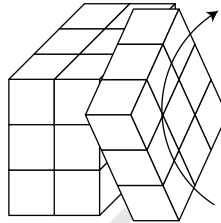
to indicate that the cubie in the *uf* position is moved to the *ru* position by the process FR.

All of the notation presented so far in this section has referred to the fixed orientation we started with. That is, the locations of the center pieces on each face have remained fixed. However, sometimes it is useful to reorient the cube – perhaps to put the Right face in the front and the Back face on the right or perhaps just to turn the cube over. But, there are several different ways to turn the cube over. We need some notation to describe this more precisely. We use another set of initials to describe the cube reorientations. We use script letters of the initials. The symbol \mathcal{U} – pronounced “script U” – will denote a clockwise quarter turn of the Up face together with the entire cube under it. The symbol \mathcal{F} will denote a clockwise quarter turn of the Front face together with the entire cube behind it. Similarly, \mathcal{R} , \mathcal{L} , \mathcal{B} , and \mathcal{D} will denote clockwise rotations of the entire cube as viewed from the Right, Left, Back, or Down faces respectively. Again we will use \mathcal{U}^1 , \mathcal{F}^1 , \mathcal{R}^1 , \mathcal{L}^1 , \mathcal{B}^1 , and \mathcal{D}^1 to denote counter-clockwise quarter turns and \mathcal{U}^2 , \mathcal{F}^2 , \mathcal{R}^2 , \mathcal{L}^2 , \mathcal{B}^2 , and \mathcal{D}^2 to indicate half turns. Notice that

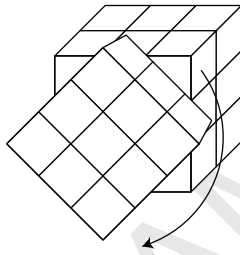
CLOCKWISE FACE TURNS



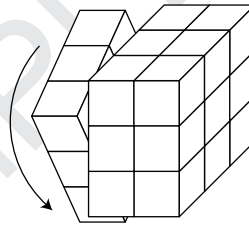
Up Clockwise



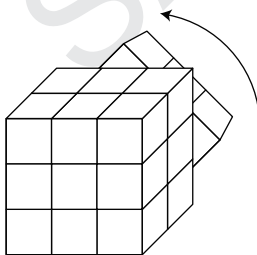
Right Clockwise



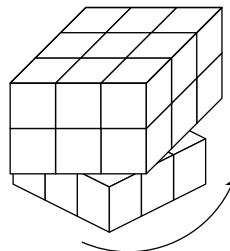
Front Clockwise



Left Clockwise



Back Clockwise



Down Clockwise

Figure 2-14

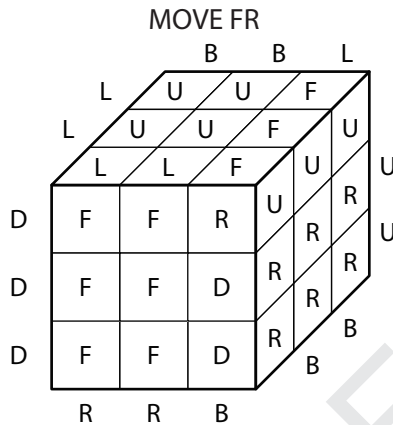


Figure 2-15

$$\mathcal{R} = \mathcal{U}\mathcal{F}\mathcal{U}^{-1}$$

This and similar equalities for the other reorientation moves show that the moves \mathcal{U} and \mathcal{F} would have been enough to reorient the cube in any way we wanted. The others still are useful abbreviations.

EXERCISES:

2.3-1 Which edge and corner locations of the cube are not shown by Figure 2-11?

2.3-2 Find a sequence of moves which accomplishes each of the following:

- a) Moves the piece in the *urf* corner to the *rdf* position.
- b) Moves the piece in the *rdf* corner to the *fur* corner.
- c) Moves the piece in the *uf* edge to the *fu* edge position – that is, flips the *uf* edge piece.

2.3-3 For each sequence of moves used to solve the previous exercise, draw a figure like Figure 2-13 showing the contents of each location on the Up, Right, and Front faces after applying each sequence to a restored cube.

2.3-4 Starting with a restored cube and using the notation presented in this chapter,

- a) List the Up-layer corner pieces.
- b) List the Down-layer edge locations.
- c) List all the pieces in the middle layer between the Up and Down faces whose location can be changed by rotating a face.
- d) List all the locations to which the *URF* piece can be moved by rotating no more than one face.
- e) List all the pieces which can be moved to the *uf* location by rotating only one face.

2.3-5 Write expressions using only \mathcal{U} and \mathcal{F} for the three orientation moves \mathcal{L} , \mathcal{B} , and \mathcal{D} .

SAMPLE